GRAĐEVINSKI FAKULTET UNIVERZITETA U BEOGRADU

Usmeni (teorijski) deo ispta iz **TEHNIČKE MEHANIKE 2**
(pismeni održan 23.06.2006.)

1. ZADATAK (5+30=35%)

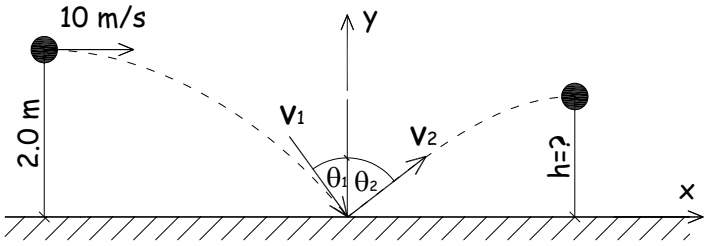
a) Definisati brzinu i ubrzanje materijalne tačke;

b) Tačka se kreće u ravni tako da je: $x=3\cdot\cos(2t)$; $y=4\cdot\sin(2t)$, gde su **x** i **y** u metrima, a **t** u sekundama. Odrediti brzinu, ubrzanje i poluprečnik krivine u trenutku $\mathbf{t=\pi/8}$.

2. ZADATAK (5+25=30%)

a) Objasniti koeficijent udara.

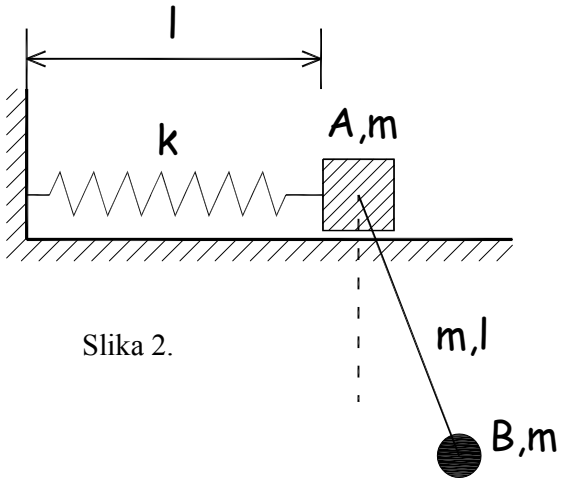
b) Materijalna tačka **M** se kreće horizontalno brzinom **10 m/s** na visini **2.0m** iznad podloge. Odrediti intenzitet i pravac brzine materijalne tačke neposredno pre i posle udara o podlogu ako je koeficijent udara $\boldsymbol{\kappa=0.7}$. Odrediti visinu maksimalnog penjenja **h** posle udara. Slika 1.



Slika 1.

3. ZADATAK (35%)

Sistem čine blok **A** mase **m**, štap **AB** dužine ***l***, mase **m** i materijalna tačka **B** mase **m**. Odrediti diferencijalne jednačine oscilovanja sistema ako se on izvede iz ravnotežnog položaja. Opruga krutosti ***k*** je dužine ***l*** u nenapregnutom stanju. Slika 2.



Slika 2.

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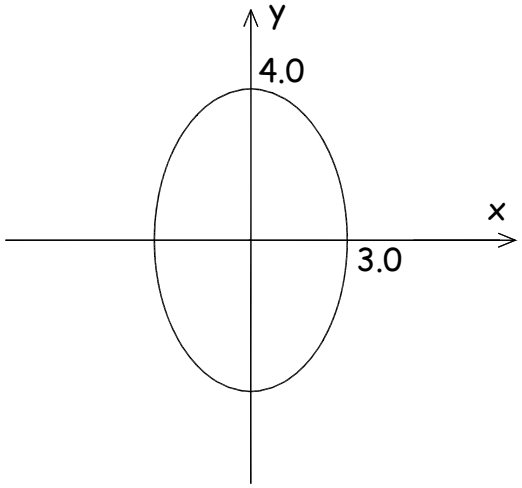
REŠENJE

1. ZADATAK

TRAJEKTORIJA:

$$\left(\frac{x}{3}\right)^2+\left(\frac{y}{4}\right)^2=1$$



BRZINA:

$$\left.\begin{aligned}\dot{x}&=-6\cdot\sin(2t)\\ \dot{y}&=8\cdot\cos(2t)\end{aligned}\right\}\ t=\frac{\pi}{8}\Rightarrow\left.\begin{aligned}\dot{x}(\pi/8)&=-6\cdot\frac{1}{\sqrt{2}}=-4.243\ \frac{m}{s}\\ \dot{y}(\pi/8)&=8\cdot\frac{1}{\sqrt{2}}=5.657\ \frac{m}{s}\end{aligned}\right\}$$

$$v=\sqrt{(-4.243)^2+(5.657)^2}=7.071\ \frac{m}{s}$$

UBRZANJE:

$$\left.\begin{aligned}\ddot{x}&=-12\cdot\cos(2t)\\ \ddot{y}&=-16\cdot\sin(2t)\end{aligned}\right\}\ t=\frac{\pi}{8}\Rightarrow\left.\begin{aligned}\ddot{x}(\pi/8)&=-12\cdot\frac{1}{\sqrt{2}}=-8.485\ \frac{m}{s^2}\\ \ddot{y}(\pi/8)&=-16\cdot\frac{1}{\sqrt{2}}=-11.314\ \frac{m}{s^2}\end{aligned}\right\}$$

$$a=\sqrt{(-8.485)^2+(11.314)^2}=14.1422\ \frac{m}{s^2}$$

POLUPREČNIK KRIVINE:

$$\vec{\tau} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{7.071}\cdot\left(-4.243\cdot\vec{i} + 5.657\cdot\vec{j}\right) = -0.600\cdot\vec{i} + 0.800\cdot\vec{j}$$

$$\vec{a} = \dot{v}\cdot\vec{\tau} + \frac{v^2}{\rho}\cdot\vec{n} = -8.458\cdot\vec{i} - 11.314\cdot\vec{j}$$

$$\vec{a}\cdot\vec{\tau} = \dot{v} = \left(-8.458\cdot\vec{i} - 11.314\cdot\vec{j}\right)\cdot\left(-0.6\cdot\vec{i} + 0.8\cdot\vec{j}\right) = -3.96 \ \ \frac{m}{s^2}$$

$$|\vec{a}| = \sqrt{\dot{v}^2 + \frac{v^4}{\rho^2}} = 14.1422 \ \ \frac{m}{s^2} \ \Rightarrow \ \frac{7.071^4}{\rho^2} = 14.1422^2 - 3.960^2 \ \Rightarrow$$

$$\rho = 3.68 \ \ m$$

Drugi način:

$$\frac{1}{\rho} = \frac{y''}{\left(1 + (y')^2\right)^{3/2}} \ \Rightarrow \rho$$

2. ZADATAK

Zakon o promeni kinetičke energije:

$$T_1 - T_0 = A_{0\to 1}$$

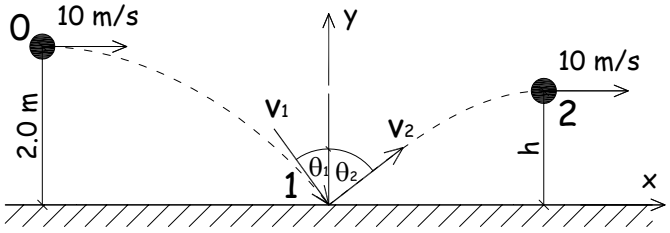
$$\frac{1}{2}m\cdot(v_1)^2 - \frac{1}{2}m\cdot(v_0)^2 = mg\cdot 2.0$$

$$(v_1)^2 = 4\cdot g + (v_0)^2$$

$$v_{1x} = 10 \ \ \frac{m}{s} = v_0$$

$$(v_{1y})^2 = 4\cdot g + (v_0)^2 - (v_0)^2 = 4\cdot g$$

$$v_{1y} = 6.264 \ \ \frac{m}{s}$$



$$tg\theta_1 = \frac{10}{6.264} = 1.5963 \Rightarrow \theta_1 = 57.93°$$

$$\kappa = -\frac{v_{2y} - 0}{v_{1y} - 0} = -\frac{v_{2y}}{-6.264} = 0.7$$

$$\Rightarrow \ \ v_{2y} = 4.3848 \ \ \frac{m}{s}; \ \ v_{2x} = 10.0 \ \ \frac{m}{s}$$

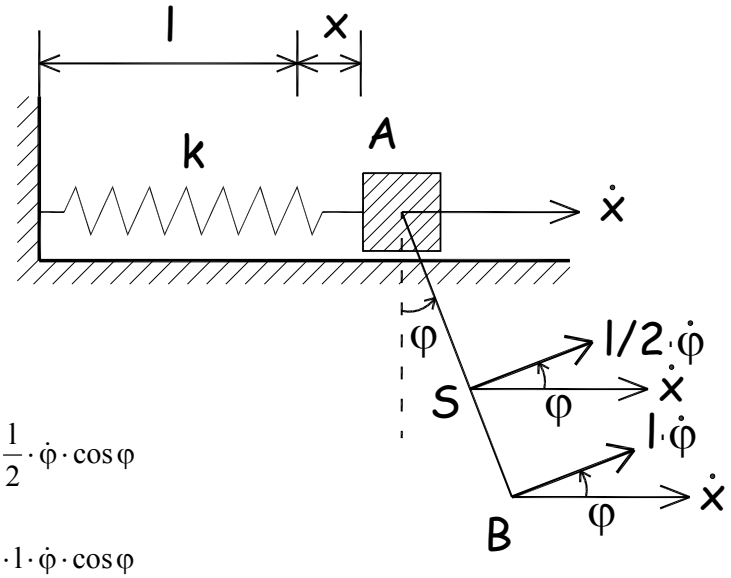
$$tg\theta_2 = \frac{10}{4.3848} = 2.286 \Rightarrow \theta_2 = 66.32°$$

$$T_1 - T_2 = A_{2\to 1}$$

$$\frac{1}{2}m\cdot(v_1)^2 - \frac{1}{2}m\cdot(v_2)^2 = mg\cdot h$$

$$\frac{1}{2}m\cdot\left(10.0^2 + 4.3848^2\right) - \frac{1}{2}m\cdot 10.0^2 = mg\cdot h \ \ \Rightarrow \ \ h = 0.979 \ \ m$$

3. ZADATAK



n=2

$q_1=x$

$q_2=\varphi$

$$v_S^2 = \dot{x}^2 + \frac{l^2}{4}\cdot\dot{\varphi}^2 + 2\cdot\dot{x}\,\frac{l}{2}\cdot\dot{\varphi}\cdot\cos\varphi$$

$$v_B^2 = \dot{x}^2 + l^2\cdot\dot{\varphi}^2 + 2\cdot\dot{x}\cdot l\cdot\dot{\varphi}\cdot\cos\varphi$$

$$T = \frac{1}{2}m\cdot\dot{x}^2 + \frac{1}{2}m\left(\dot{x}^2 + \frac{l^2}{4}\cdot\dot{\varphi}^2 + \dot{x}\cdot l\cdot\dot{\varphi}\cdot\cos\varphi\right) + \frac{1}{2}\cdot\frac{1}{12}m\cdot l^2\cdot\dot{\varphi}^2 + \frac{1}{2}m\left(\dot{x}^2 + l^2\cdot\dot{\varphi}^2 + 2\cdot\dot{x}\cdot l\cdot\dot{\varphi}\cdot\cos\varphi\right)$$

$$T = \frac{3}{2}m\cdot\dot{x}^2 + \frac{2}{3}m\cdot\dot{\varphi}^2 + \frac{3}{2}m\cdot l\cdot\dot{x}\cdot\dot{\varphi}\cdot\cos\varphi$$

$$\frac{\partial T}{\partial\dot{\varphi}} = \frac{4}{3}\cdot m\cdot\dot{\varphi} + \frac{3}{2}m\cdot l\cdot\dot{x}\cdot\cos\varphi$$

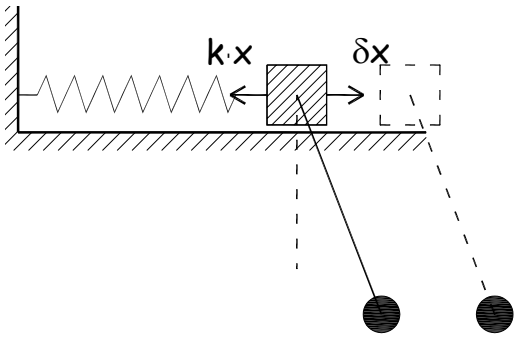
$$\frac{\partial T}{\partial\dot{x}} = 3m\cdot\dot{x} + \frac{3}{2}m\cdot l\cdot\dot{\varphi}\cdot\cos\varphi$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{x}}\right) = 3m\cdot\ddot{x} + \frac{3}{2}m\cdot l\cdot\ddot{\varphi}\cdot\cos\varphi - \frac{3}{2}m\cdot l\cdot\dot{\varphi}^2\cdot\sin\varphi$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}}\right) = \frac{4}{3}\cdot m\cdot\ddot{\varphi} + \frac{3}{2}m\cdot l\cdot\ddot{x}\cdot\cos\varphi - \frac{3}{2}m\cdot l\cdot\dot{x}\cdot\dot{\varphi}\cdot\sin\varphi$$

$$\frac{\partial T}{\partial x} = 0;\quad \frac{\partial T}{\partial\varphi} = -\frac{3}{2}m\cdot l\cdot\dot{x}\cdot\dot{\varphi}\cdot\sin\varphi$$

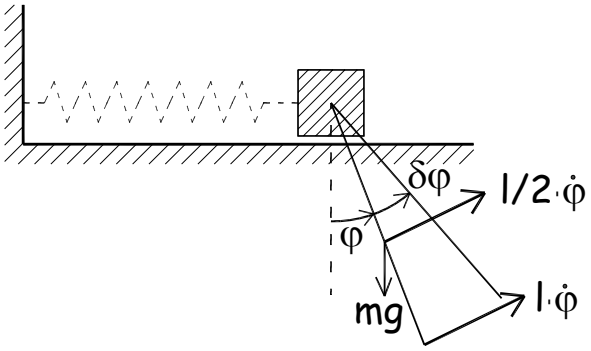
GENERALISANE SILE:



Q_x=?; $\delta x \neq 0$; $\delta\varphi = 0$;

$$\delta A = Q_x \cdot \delta x$$
$$Q_x \delta x = -k \cdot x \cdot \delta x$$
$$Q_x = -k \cdot x$$

Q_φ=?; $\delta x = 0$; $\delta\varphi \neq 0$;



$$Q_\varphi \delta\varphi = -mg\frac{1}{2}\sin\varphi \cdot \delta\varphi - mgl\sin\varphi \cdot \delta\varphi$$
$$Q_\varphi = -\frac{3}{2}mg\frac{1}{2}\sin\varphi$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i$$

$$3m\ddot{x} + \frac{3}{2}ml\ddot{\varphi}\cos\varphi - \frac{3}{2}ml\dot{\varphi}^2\sin\varphi = -k \cdot x$$
$$\frac{4}{3}m\ddot{\varphi} + \frac{3}{2}m\ddot{x}\cos\varphi = -\frac{3}{2}mgl\sin\varphi$$